APPENDIX F. ASPECT RATIO CALCULATIONS FOR BUILDINGS WITH COURTYARDS

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F.1 COURTYARD ASPECT RATIO DETERMINATION

A single-story rectangular building with a courtyard with the distance from the outside wall to the courtyard of a constant depth D, width W and Aspect Ratio R has the following formulas for Area and Perimeter:

$$
\begin{aligned}
\text{Perimeter} &= \text{outside perimeter} + \text{courtyard perimeter} \\
&= 2\,W\,(R+1) \quad + \quad 2\,(RW-2D + W-2D) \\
&= 4RW + 4W - 8D \\
&= 4\,(RW + W - 2D)
\end{aligned}
$$

$$
\begin{aligned}
\text{Area} &= 2\,(D)\,(W-2D) + 2\,(D)\,(RW-2D) + 4\,D^2 \\
&= 2\,D\,(RW + W - 2D)
\end{aligned}
$$

And the ratio of the perimeter to area (P/A) becomes simply

$$P/A = 2\,/\,D$$

This is important. The ratio of the Perimeter to Footprint area for these buildings is a constant once the depth D is known.

F.2 MAPPING TO A RECTANGULAR BUILDING WITHOUT A COURTYARD

If we want to produce a rectangular building with the same ratio of perimeter to area, we start with the ratio of perimeter P' to area A' in any rectangular building of width W' and aspect ratio R'

$$
\begin{aligned}
P' &= 2(W+RW') \\
&= 2W'(R'+1)
\end{aligned}
$$

$$A' = (R'W')(W')$$

$$P/A = 2\,(R'+1)\,/R'\,W'$$

Equating P'/A' with P/A we get

$$2\,(R'+1)\,/(R'W') = 2\,/\,D$$

We do not know R’ or W’ currently, but we do know that we want the floor area of the building with the courtyard to be the same as that of our rectangular (non courtyard) building. Hence, $W' = (A/R')^{0.5}$, and substituting

$$2\,(R'+1)/(A^{0.5}R'^{0.5}) = 2/D$$
$$(R'+1)/R'^{0.5} = (A^{0.5}/D)$$

$$R'^2 + 2R' + 1 = AR'/D^2$$
$$R'^2 + (2-A/D^2)R' + 1 = 0$$

And solving the quadratic in R

$$R' = \frac{-(2-A/D^2) +/- [(2-A/D^2)^2 - 4]^{0.5}}{2}$$

$$R' = \frac{-(2-A/D^2) +/- [(4-4A/D^2+A^2/D^4 - 4]^{0.5}}{2}$$

$$R' = \frac{A/D^2 - 2 +/- [(A^2/D^4 - 4A/D^2]^{0.5}}{2}$$

The two solutions correspond to R = L/W and R = W/L. We want the larger, greater than 1.0 value for R’. If we further substitute parameter Z for A/D^2

$$R' = \frac{A/D^2 + [(A^2/D^4 - 4A/D^2]^{0.5} - 2}{2}$$

$$R' = \frac{Z + [(Z^2 - 4Z]^{0.5} - 2}{2}$$

$$R' = \frac{Z + \left(Z^2 - 4Z\right)^{0.5} - 2}{2}$$

at the lower limit (a square building with infinitesimal courtyard) $A = 4D^2$ and Z = 4 and R = R’ = 1, which is what you would expect (start with a square, end with a square building).

At the upper limit, Z goes to infinity and R’ ~ Z, which is also what you would expect from a thin ribbon of building with courtyard.